Forecasting in a DSGE Model for Turkey: A Bayesian Approach

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ABSTRACT

In this study, a Small Open Economy theoretical model based on Galí and Monacelli (2005) which is a version of Dynamic Stochastic General Equilibrium (DSGE) model will be combined with a Vector Auto Regression (VAR) model and the resulting DSGE-VAR model will be used to make long term projections through Bayesian estimations for the Turkish economy. DSGE models are generally used in the analyses of short term macroeconomic policy within the New Keynesian framework. The long term projections are made generally via the VAR models. In this study, the two approaches will be combined and projections will be made by using a VAR model starting from a structural DSGE model for Turkey. Because in literature, there are not many long term projections and estimations made within such a framework, this study will contribute the literature. The estimation and forecasting results are obtained for Turkey's 2023 GDP and per capita GDP. The results show that those 2023 targets will not be met. So if Turkey is serious and insistent on these targets, these study may be warning to policymakers and the public to take the necessary measures when there is still enough time and opportunity.

Keywords: DSGE-VAR, SOE, Bayesian Methods, Projection

1. INTRODUCTION

The Turkish economy has almost experienced a continuous growth process since the first quarter of the year 2002, except for the year 2009 in which the impact of global financial crisis was felt. As a result of this growth period, Turkey has become the 17th or 18th largest economy in the world. [1]Following this remarkable economic performance, Turkey has set some big targets for 2023 such as having GDP of [2] trillion US dollars or per capita GDP of 25 thousand US dollars.

In this study, the Turkish economy will be modeled and investigated through a Small Open Economy (SOE) theoretical model based on Galí and Monacelli (2005). This model is a version of Dynamic Stochastic General Equilibrium (DSGE) model.The projections in this study will be made through Bayesian estimations of a DSGE-VAR (or DSGE-VAR(λ)) model that is obtained from the combination of a structural DSGE model with a Vector Auto regression (VAR) model. Therefore, the DSGE-VAR (λ) model establishes a balance between the statistical representation (VAR) and the economic requirement (DSGE). So here, the DSGE-VAR or DSGE-VAR (λ) model that will be estimated by using Bayesian methods represents a combination of our SOE model with a VAR model.[2]DSGE models are generally used in the analyses of short term macroeconomic policy within the New Keynesian framework. The long term projections are made generally by the VAR models. In this study, these two approaches will be combined and projections will be made by using a VAR model starting from a structural DSGE model. Because in literature, there are no long term projections and estimations made within such a framework at least to our knowledge, this study will contribute the literature.

So, in such a framework this study investigates whether Turkey would be able to meet its 2023 targets or not if the trends and dynamics that have been experienced during the last decade continue. Therefore, in this study, we will just specifically look at the feasibility of GDP and per capita GDP targets of Turkey in 2023.

In addition to the theoretical and methodological contributions, this study could provide information and guidance to policymakers in their way of reaching the targets. So, if they are serious on those targets, and such studies arrive at a conclusion

1 According to the IMF, World Bank and UN ranking based on nominal GDPs in 2014 and 2015, Turkey is the 18th in the list. However, according to GDP calculated by purchasing power parity (PPP), Turkey is 17th in the world GDP ranking lists.

2 The hyper parameter λ is interpreted as the weight placed on both the VAR and the DSGE parts of the DSGE-VAR(λ). $\lambda \to \infty$: DSGE-VAR(λ) $\to$ DSGE; $\lambda \to 0$: DSGE-VAR(λ) $\to$VAR

that these targets are not attainable, then the attention of policymakers and the public can be captured when there is still enough time to take the necessary measures. Another contribution could be introducing of using Bayesian methods and techniques in Turkey in the context of long term projections, where there are not enough studies using this state-of-art estimation method.

2. THE LITERATURE

2.1 DSGE Models

We will use a small open economy (SOE) model which is a variant of the dynamic stochastic general equilibrium (DSGE) model in this study. This is an open economy version of the standard model used in New Keynesian framework (see Woodford, 2003) developed first by Galí and Monacelli's (2005).Its simplification by Lubik and Schorfheide (2007) has become standard and vastly used in the literature. DSGE models are micro founded optimization-based models that have become very popular in macroeconomics over the past 25-30 years. DSGE models are the models with a high degree of theoretical coherence and derived from the first principle by explicitly modeling the household and firm behavior as well as the monetary authority's reaction function. In these models, decision rules of economic agents are derived from assumptions about agents' preferences and production technologies and some fundamental principles such as inter temporal optimization, rational expectations, and competitive equilibrium. Thus, they are robust against the well-known Lucas (1976) critique, and a good model for policy analysis. However in such models, the functional forms and parameters of equations that describe the behavior of economic agents are tightly restricted by optimality and equilibrium conditions. But, likelihood functions for such empirical models with a strong degree of theoretical coherence tend to be more restrictive than likelihood functions associated with a theoretical models. However, a challenge arises if the data favor the a theoretical model. Since the a theoretical model generates more accurate forecasts, but a theoretically coherent model is required for the analysis of a particular economic policy.

In literature DSGE models could be estimated by different methods. Clarida, Galíve Gertler (2000) used GMM. Orphanides (2001) or Ball and Tchaidze (2002) used OLS methods but to avoid endogeniety bias they made implausible identification assumptions. Fuhrer and Moore (1995), Leeper and Sims (1994), Kim (2000) used full-information maximum likelihood estimation (MLE) method. However, one problem in the estimation of DSGE models by MLE is absurd parameter estimates, that is, estimates of structural parameters by MLE are often at odds with additional information or observations. Because of such problems DSGE models have been estimated by Bayesian methods recently. Likelihood-based Bayesian estimations of DSGE models have started with the studies of Landon-Lane (1998), DeJong et all. (2000), Schorfheide (2000) and Otrok (2001).An analysis and estimation of a DSGE model by Bayesian methods in a closed economy framework was performed by An and Schorfheide (2007).In open-economy literature, Lubik and Schorfheide (2007) used Bayesian estimation methods in a SOE framework to see the effects of exchange rates movement on the monetary policies of some central banks (to investigate the hypothesis whether central banks do respond to exchange rates).Lubik and Schorfheide (2006), Rabanal and Tuesta (2006), Walque and Wouters (2004) used Bayesian methods in the estimations of multi-country DSGE models.

2.2 VAR Models

While DSGE models provide a complete multivariate stochastic process representation for the data, simple models impose very strong restrictions on actual time series and are in many cases rejected against less restrictive specifications such as Vector Auto Regressions (VAR).A natural alternative to DSGE models in dynamic macroeconomics is a VAR model, because linearized DSGE models, at least approximately, can be interpreted as restrictions on a VAR representation. Thus, instead of estimating a structural model of the economy, we can directly estimate a model by using observable variables and data without having any restrictions, as in a VAR model. VARs are linear time-series models, designed to capture the joint dynamics of multiple time series.

Therefore, VAR models are usually used in long-term estimations and making future projections. Sims (1980) proposed VARs to replace large-scale macro econometric models (inherited from the 1960s), which impose incredible restrictions. Since then, VARs have been used for macroeconomic forecasting and policy analysis to investigate the sources of business-cycle fluctuations and to provide a benchmark against which modern dynamic macroeconomic theories can be evaluated. The equilibrium law of motion of many dynamic stochastic equilibrium models can be well approximated by a VAR. There are some problems in the estimations and forecasting with VAR models. Since the VAR parameter space is generally much larger than the DSGE model parameter space, the specification of a prior distribution for the VAR parameter becomes very important and requires careful attention. A VAR with a prior that is very diffuse is likely to be rejected even against a misspecified DSGE model (An and Schorfheide (2007)).[3]

3. In a more general context this phenomenon is often called Lindley's paradox.

3. THE MODEL AND THE ESTIMATION METHOD

3.1 DSGE-VAR and Bayesian Approach

In empirical studies the guidance of theory is crucial. However, since theoretical models have been getting more complicated, complex and very specific in macroeconomics, this guidance in a formal statistical framework has become more difficult. Bayesian estimation methods provide a statistical framework to overcome such difficulties and take formally uncertainties in model parameters into account. In Bayesian inference, prior distributions for parameters are updated by sample information contained in the likelihood functions to form posterior distributions. In a Bayesian framework, prior distributions are important. The prior can enable to use information that is not contained in the estimation sample. Thus, to the extent that the prior is based on non sample information, it provides the ideal framework for combining different sources of information and thereby sharpening inference, and obtaining more correct and consistent estimations in macro econometric analysis. Then, posterior distributions can be obtained to measure parameters' uncertainty, and used in making political analysis, estimations, and future forecasting.

Thus, with a DSGE-VAR model in Bayesian framework, since the likelihood function is reweighted by a prior density that can bring the information to the model that is not contained in the estimation sample, more reasonable and consistent estimations can be obtained. In a DSGE-VAR model, probability distributions for parameters are determined by a DSGE model that models the economy theoretically. These distributions are used as prior distribution in VAR to obtain estimations and projections that are consistent with the assumed economic model. Then, the posterior distribution of DSGE-VAR model is derived from combining a VAR likelihood function with the DSGE priors. The DSGE priors here are used instead of the Minnesota-styled priors that are usually used in the Bayesian VAR (BVAR).

DSGE-VAR model obtained by combining DSGE models with Bayesian VARs first proposed by Ingram and Whiteman (1994), and further developed by Del Negro and Schorfheide (2004) to improve forecasting and monetary policy analysis with VARs. Then the framework has been extended to a model evaluation tool in Del Negro et al. (2007).The main point in these studies is to determine the moments of the prior distribution of the VAR parameters by using a DSGE model. DSGE-VAR approach was designed to improve forecasting and monetary policy analysis with VARs, and some studies find that this model can compete in forecasting with BVARs based on the Minnesota prior (Del Negro and Schorfheide (2006, 2009)).Since unlike the BVAR, where the so-called Minnesota priors are used to tilt the estimates toward random walks in the parameter space, the DSGE-VAR model uses the artificial data generated from the DSGE to tilt the estimates toward the region of the parameter space. This would produce better and more consistent estimation results when theoretically strong DSGE models are used.

3.2 The Small Open Economy (SOE) Model

A SOE-DSGE model includes an open economy IS curve that represents the production side of the economy, a new Keynesian Phillips curve that represents inflation dynamics, a real exchange rate equation, and a monetary policy rule. Thus, the SOE model that will be used in this study can be described by the following equations.

The consumption Euler equation showing the supply side of the economy can be rewritten as an open economy IS-curve:

$$y_t=E_t\, y_{(t+1)}-[\tau+\alpha(2-\alpha)(1-\tau)](R_t-E_t\, \pi_{(t+1)})-\rho_z\, z_t$$

$$-\alpha\left[\tau+\alpha(2-\alpha)(1-\tau)\right]E_t\, \Delta q_{t+1}+\alpha(2-\alpha)\frac{1-\tau}{\tau}E_t\, \Delta y^*_{t+1}$$

where $0 < \alpha < 1$ is the import share, τ is the intertemporal substitution elasticity, y is aggregate output, and π is the CPI inflation rate. The terms of trade, q is defined as the relative price of exports in terms of imports. y* is exogenous world output, and z is the growth rate of an underlying non-stationary world technology process A.[4]

Optimal price setting of domestic firms leads to the open economy Phillips curve:

$$\pi_t=\beta E_t\pi_{t+1}+\alpha\beta E_t\, \Delta q_{t+1}-\alpha\, \Delta q_t+\frac{\kappa}{\tau+\alpha(2-\alpha)(1-\tau)}(y_t-\bar{y}_t)$$

4. In order to guarantee stationary of the model, all real variables are therefore expressed in terms of percentage deviations from A.

where $\overline{y}$ is potential output in the absence of nominal rigidities. The slope coefficient κ is a function of underlying structural parameters, such as labor supply and demand elasticities and parameters capturing the degree of price stickiness.

The nominal exchange rate e is included into the model through the definition of the CPI by assuming that relative PPP holds:

$$\pi_t = \Delta e_t + (1-\alpha)\Delta q_t + \pi_t^*$$

where π* is an unobserved world inflation shock, and may also be interpreted as the misspecification, or deviations from PPP.

Monetary policy is described by a Taylor-type interest rate rule, where the central bank adjusts its instrument in response to movements in CPI inflation, output, and nominal exchange rate:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left[\psi_1\pi_t + \psi_2 y_t + \psi_3\Delta e_t\right] + \varepsilon_t^R$$

Where ψs represents monetary policy coefficients, and is a smoothing term that accounts the persistence in nominal interest rates.

The terms of trade can be determined endogenously as the relative price that clears international goods markets. However, estimation of the fully structural model turned out to be problematic as explained in Lubik and Schorfheide (2007).For most specifications, numerical optimization routine had difficulties finding the maximum of the posterior density. Whenever optimization did converge, implausible parameter estimates and low likelihood values are obtained. The apparent reason is that endogenous equation would imply a tight link between the terms of trade and output growth that the estimation procedure attempts to match. This creates a conflict with output and inflation dynamics as governed by the IS-equation and the Phillips-curve, which can at best only be resolved at the cost of implausible estimates. To overcome such difficulties, a law of motion for the growth rate of the terms of trade can be added to the system of equations given above:

$$\Delta q_t = \rho_q \, \Delta q_{(t-1)} + \varepsilon_{(q,t)}$$

These five equations given above form a linear rational expectations model. Here it will be assumed that the variables representing the world π_t^* and y_t^* evolve according to univariate AR(1) processes with autoregressive coefficients $\rho_{\pi*}$ and ρ_{y*}, respectively. The innovations of the AR(1) processes are denoted by $\varepsilon_{\pi*}$ and ε_{y*}.This rational expectations model can be solved by different methods. Linear approximation methods are very popular.Since a log-linearized DSGE model with rational expectations can be put in a state-space form, where the observed variables are linked to the model variables through the measurement equation. At the same time, the state equation provides the reduced form of the DSGE model, mapping current variables to their lags and the i.i.d. shocks. The reduced form is obtained by solving for the expectation terms in the structural form of the model using a suitable numerical technique. The most common methods are Anderson and Moore's (1985) AiM algorithm, Klein (2000), and Sims (2002).[5]If a unique convergent solution is available, the Kalman filter can be applied to compute the value of the log-likelihood function.

3.3 Estimation in Bayesian Framework

After solving the DSGE model, a parameter vector that needs to be estimated is formed. This vector is composed of unknown model parameters, policy rule parameters, and parameters for the shocks. Under the assumption that all the structural shocks are normally distributed and uncorrelated with each other at all leads and lags we can obtain a joint probability distribution for the endogenous model variables. The solution of the rational expectations system takes the form

$$s_t = \Phi(s_{(t-1)}, \epsilon_t; \theta)$$

The variables in the rational expectations system are represented by the vector st.From an econometric perspective, st can be viewed as a (partially latent) state vector in a non-linear state space model and the above equation is the state transition equation. Here, is the vector of innovations and the structural parameters are collected in the vector θ that are the parameters to estimate in the model. The model is completed by defining a set of measurement equations that relate the elements of st to a vector of observations, Y.

The unknown parameters will be estimated by Bayesian techniques. A Bayesian approach is in principle easy. Parameters are given values through their posterior distribution, which is linked to prior information and the observed data through Bayes theorem. From Bayes theorem we know that the posterior distribution of θ, denoted by p(θ|Y), is given as

5. see also Blanchard and Kahn (1980), Anderson (2008, 2010), Christiano (2002), King and Watson (1998)

$$p(\theta|Y) = \frac{p(Y|\theta)\,p(\theta)}{p(Y)}$$

where p(θ) is the prior density, p(Y|θ) is the density function for a random data matrix Y conditional on θ, and p(Y) is the marginal data density that is defined as

$$p(Y)=\int p(Y|\theta)\;p(\theta)d\theta=\int L(\theta|Y)\;p(\theta)d\theta$$

As can be seen from the above equation, the density function p(Y|θ) can be represented by the likelihood function *L(Y|θ)* associated with the DSGE model and Yis the vector of observables.[6]In a Bayesian framework, the likelihood function is reweighted by a prior density. If the likelihood function peaks at a value that is at odds with the information that has been used to construct the prior distribution, then the marginal data density of the DSGE model, will be low compared to, say, a VAR

If the pure Bayesian approach is used, a prior distribution for each parameter is assigned, and the data are used to update these priors through the likelihood function of the DSGE model. Then, by using Bayes theorem the posterior distributions can be obtained. In a Bayesian framework, this likelihood function can be used to transform a prior distribution for the structural parameters of the DSGE model into a posterior distribution. This posterior is the basis for inference and decision making.

However, in the DSGE-VAR estimation method, the DSGE model is used to determine the moments of the prior distribution of the VAR parameters using a normal/inverted Wishart distribution. Thus, possible distributions of parameters are determined by using a DSGE model that theoretically represents the economy. These distributions are used as priors in VAR estimation, so estimation and forecasting results that are consistent with the assumed economy could be obtained. That is, dummy observations priors are obtained by the DSGE model, and then these DSGE priors are used to weigh the VAR likelihood function in order to derive the posterior distribution. These posterior distributions are used for posterior sampling or posterior mode estimation to obtain estimation or forecasting results. The DSGE priors here are used instead of the Minnesota-styled priors that are usually used in the BVAR. It is found that this model can compete in forecasting exercises with BVARs based on the Minnesota prior.

The DSGE-VAR (λ) shows an equilibrium between economic requirement represented by a theoretical model (DSGE) and statistical representation in an empirical framework (VAR) depending on the optimum value of λ. Since the empirical performance of the DSGE-VAR(λ) procedure crucially depends on the weight placed on each part, a data-driven procedure to determine optimum λ will be used (An and Schorfheide (2007)).A natural criterion for the choice of λ in a Bayesian framework is the marginal data density

$$p\lambda\,(Y)=\int p\lambda\,(Y|\theta)\;p(\theta)d\theta$$

For computational reasons we restrict the hyper parameter to a finite grid of λs. If one assigns equal prior probability to each grid point then the normalized $p_\lambda(Y)$'s can be interpreted as posterior probabilities for λ. Then the optimum value for λ can be obtained from:

$$\hat{\lambda} = \arg\max_{\lambda\in\Lambda} p_\lambda(Y)$$

4. THE ESTIMATION RESULTS

In this study, seasonally adjusted quarterly data set for Turkey will be used. The data set includes nominal GDP, inflation rates, short-term interest rates, nominal exchange rates, and terms of trades. In estimations and forecasting, growth rates of output will be used; inflation will be taken as the change in CPI; exchange rates will be either trade-weighted nominal exchange rate or simply dollar exchange rates. Since the changes in exchange rates and terms of trades will be used in estimations, log differences of those variables will be taken.

6. In this marginal density of the datawith θ ∈ Θ being the support of θ. Since p(Y) is a constant when Y has been realized we know from the Bayes theorem that the posterior density of θ is proportional to the product p(Y|θ)p(θ). Hence, if we can characterize the distribution of this product we would know the posterior distribution of θ. For complex models like those belonging to the DSGE family this characterization is usually not possible. Methods based on Markov Chain Monte Carlo (MCMC) theory can instead be applied to generate draws from the posterior.

Two data sets for Turkey are used in forecasting, one covers the period 1998:2-2015:4, and the other period 2002:1-2015:4. Although the first set has more observations, it also includes the effects of big crisis in Turkey in 2001. However, the second set covers observations after the crisis, so it represents a period in which Turkey is more stable. Thus, it might be thought that a projection using the second set may reflect the current dynamics of Turkey more and also carry these into the future in a better way.

The model is estimated through Bayesian estimation methods described above and quarterly projection results are obtained. Table 1 shows the annual results of projection calculated from quarterly results when 1998-2015 data set is used. Also, it tabulates three confidence bands of 50%, 70% and 90% calculated for this projection. Such as the values for 90% confidence band indicates that projection results would be in this interval with 90% probability. Figure 1 shows quarterly projection results graphically that are used to obtain annual results given in Table 1. According to these results, nominal GDP of Turkey in 2023 is projected to be 3.8 trillion TL. Also it can be expected that GDP in 2023 would be between 3.59 trillion and 3.99 trillion liras with 90% probability. The projection results obtained when 2002-2015 results are used are given in Table 2 and Figure 2. They indicates that the higher projection results are obtained with the data set that excludes the 2001 crisis, and 2023 GDP is projected to be 4.72 trillion TL. Similar projections are also implemented in US dollars and the results are given in Table 3, Table4, and Figure 3 and Figure 4. According to these projections Turkey's GDPs in 2023 is found to be 942.4 billion dollars and 947.2 billion dollars for the two data sets. These findings show that Turkey's GDP target of reaching 2 trillion dollars in 2023 seems to be far from achieving.

Per capita GDP projections are given in Table 5 and Table 6. According to these results GDP per capita in 2023 is projected to be around 11.200 dollars, and 90% confidence band results shows that this figure could be realized as highest as 12.000 dollars with probability of 90%. Therefore, these findings show that it is difficult for Turkey to reach the per capita target level of 25.000 dollars in 2023.

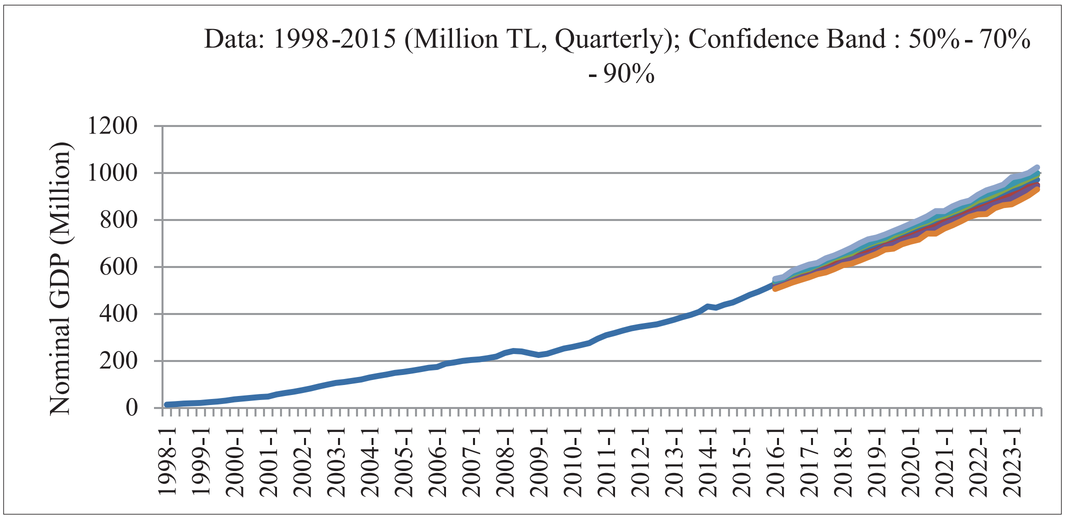

Figure 1: Nominal GDP Projection with 1998-2015 Data

Table 1: GDP projection with 1998-2015 Data - TL

Year	Nom.GDP (1000 TL)	Confidence band					
2015	1952724009	50%		70%		90%	
2016	2193749416	2150714971	2232520925	2134096574	2253937948	2104694703	2287879265
2017	2406717216	2362485652	2456648353	2331234081	2477383867	2294364246	2515110036
2018	2625544162	2575736488	2680796985	2544951019	2709645749	2491961424	2766325150
2019	2847140174	2783904122	2909063081	2759677984	2941297862	2705912849	2986203550
2020	3074239791	3009787960	3143449030	2970692811	3174796445	2909766651	3238328122
2021	3307628174	3242805705	3363672744	3207626674	3401163898	3151116033	3454044063
2022	3547673244	3473454840	3614624077	3435794156	3666190783	3364553222	3723388383
2023	3795193990	3712127398	3870907018	3673746596	3910672754	3589206313	3995732916

5. RESULTS

The projection results show that the 2023 targets of Turkey of having GDP of 2 trillion dollars and per capita GDP of 25.000 dollars are far from reaching and seem to be unattainable with the current dynamics and trends. The reasons to be far from these targets today could be explained by large depreciation of Turkish currency of lira against US dollar in 2014 and 2015. Because, by excluding 2014 and 2015 data and making projections with the data set until 2013, we have obtained the GDP of 1.5 trillion dollars and per capita GDP of 17-18 thousand dollars projections for 2023. Although they are still below the target levels, they are not very far from those targets as the figures given above.

Table 2: GDP projection with 2002-2015 Data - TL

Year	Nom.GDP (1000 TL)	Confidence band					
2015	1952724009	50%		70%		90%	
2016	2262066340	2221037944	2304509157	2199508443	2325397912	2163984989	2371761603
2017	2598143283	2550010624	2648562648	2517288533	2678684905	2466901117	2723968058
2018	2944276252	2890178910	2997265375	2851333622	3034014474	2806117803	3075747459
2019	3288115690	3213169550	3360336406	3172140651	3397247470	3110472783	3471206420
2020	3631337085	3545853333	3716175017	3508599029	3753194814	3440753719	3826279266
2021	3980038870	3894526203	4067389578	3855056734	4110047341	3783519263	4178206433
2022	4339937130	4244517251	4439772203	4183699624	4489845853	4118373300	4560201075
2023	4716567615	4624202565	4809816086	4582123040	4869340126	4475340483	4928979127

Table 3: GDP projection with 1998-2015 Data - USD

Year	Nom.GDP (1000 USD)	Confidence band					
2015	721051070	50%		70%		90%	
2016	708891564	678250146	735392054	666045267	752679076	640813019	779743837
2017	728773540	691806891	769305255	669084167	788727180	645364937	823308215
2018	757975618	725538167	793373864	701471104	812952544	669841378	842973334
2019	793173789	758482702	828043276	736589167	850595994	699585683	888667746
2020	829083127	788854812	866866464	770221475	887260258	740038309	930644355
2021	865876586	827982218	902141266	803533619	925187654	767524204	967598530
2022	903681691	859442943	950854956	833339864	974778373	800430358	1016097404
2023	942447762	901729705	981916419	879088358	1011907123	840976999	1061962832

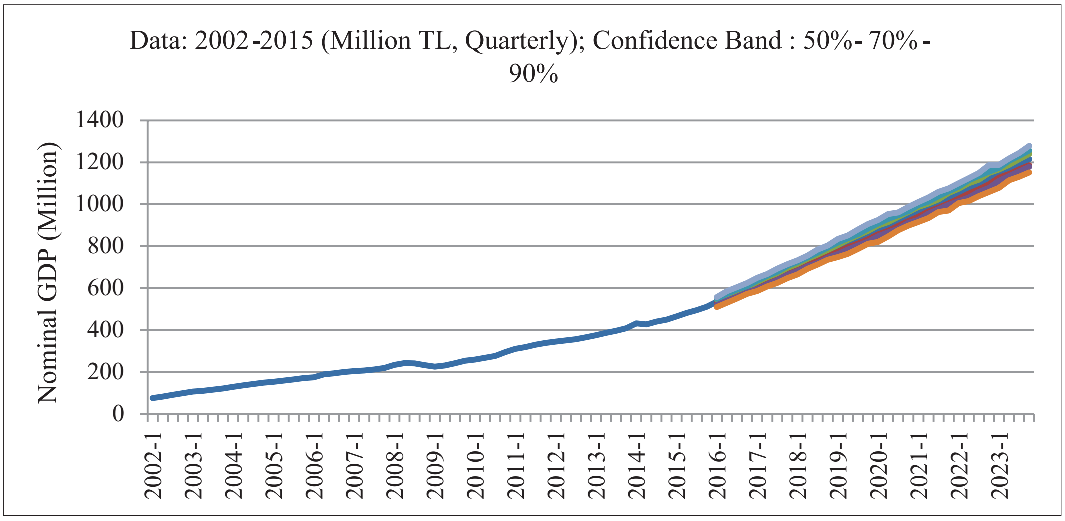

Figure 2: Nominal GDP Projection with 2002-2015 Data

Table 4: GDP projection with 2002-2015 Data - USD

Year	Nom.GDP (1000 USD)	Confidence band					
2015	721051070	50%		70%		90%	
2016	728438318	701942363	752182942	689750070	768195146	667502612	793611342
2017	756947587	727078666	784560522	713023262	802622435	689926856	829846759
2018	785647730	757668315	816295276	738085477	832364933	717309822	850074051
2019	818291180	788375418	845712519	774947268	866035139	749262163	895613375
2020	850482239	820002538	879577859	803711740	896273246	776195324	927968454
2021	882777091	849155780	915936565	831598994	936885991	801566996	963900320
2022	914945081	882394941	948876084	862670724	966564519	829068777	998334015
2023	947180046	913729768	982827589	897139306	1001711473	866082637	1026237387

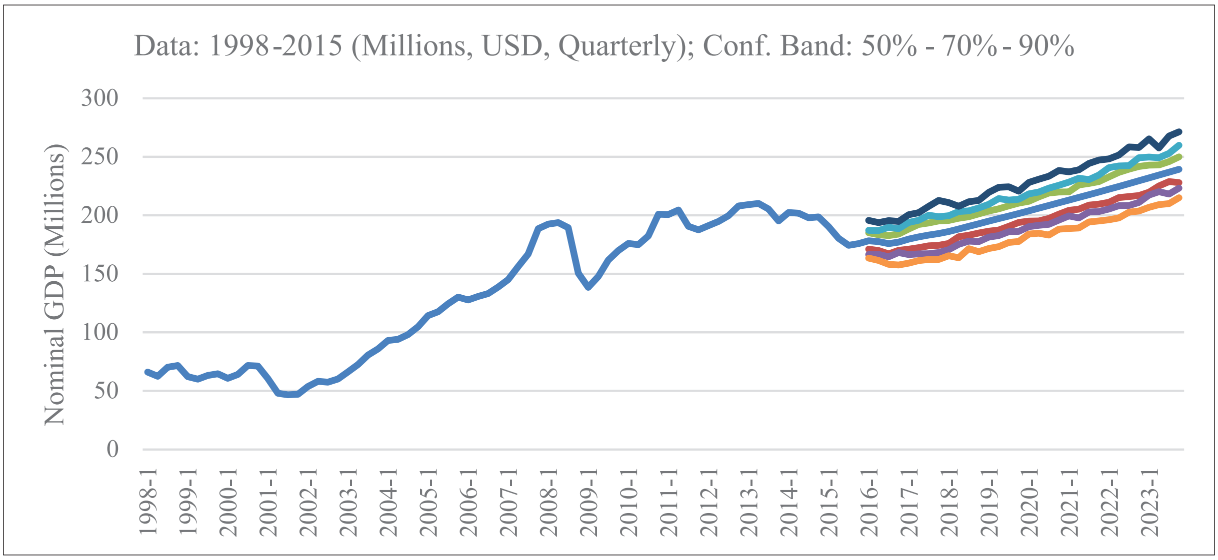

Figure 3: Nominal GDP Projection with 1998-2015 Data

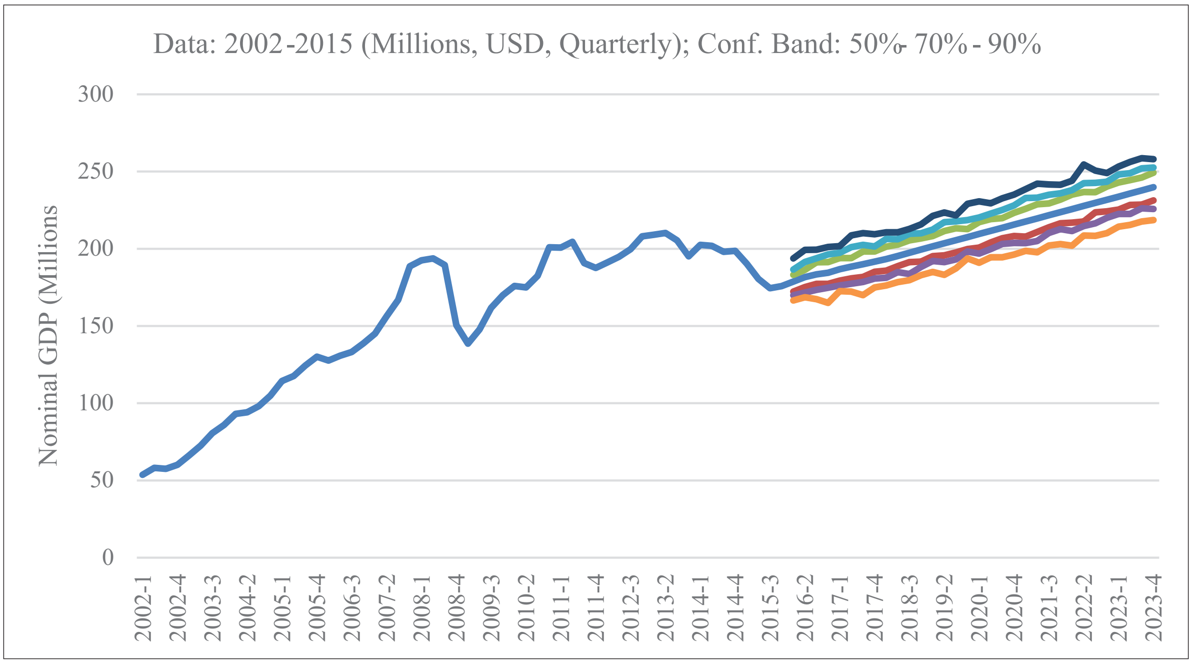

Figure 4: Nominal GDP Projection with 2002-2015 Data

Table 5: Nominal GDP per Capita projection with 1998-2015 Data - USD

Year	Nominal GDP per capita 1998-2015 (thousand USD)	Confidence band					
2015	9157.24	50%		70%		90%	
2016	8977.21	8589.18	9312.81	8434.62	9531.73	8115.09	9874.47
2017	9136.39	8672.95	9644.52	8388.09	9888.01	8090.73	10321.54
2018	9409.85	9007.16	9849.30	8708.38	10092.36	8315.72	10465.05
2019	9753.55	9326.96	10182.33	9057.73	10459.66	8602.71	10927.82
2020	10101.31	9611.18	10561.65	9384.16	10810.12	9016.41	11338.70
2021	10455.39	9997.82	10893.29	9702.61	11171.57	9267.80	11683.68
2022	10817.34	10287.79	11382.02	9975.33	11668.39	9581.39	12162.99
2023	11186.71	10703.39	11655.20	10434.64	12011.18	9982.27	12605.34

Table 6: Nominal GDP per Capita projection with 2002-2015 Data - USD

Year	Nominal GDP per capita 2002-2015 (thousand USD)	Confidence band					
2015	9157.24	50%		70%		90%	
2016	9224.75	8889.21	9525.45	8734.81	9728.22	8453.08	10050.08
2017	9489.60	9115.14	9835.77	8938.94	10062.21	8649.38	10403.51
2018	9753.39	9406.04	10133.86	9162.93	10333.36	8905.01	10553.21
2019	10062.41	9694.54	10399.61	9529.42	10649.51	9213.57	11013.23
2020	10362.03	9990.68	10716.52	9792.19	10919.94	9456.94	11306.10
2021	10659.47	10253.49	11059.87	10041.50	11312.83	9678.86	11639.02
2022	10952.17	10562.53	11358.33	10326.43	11570.07	9924.20	11950.36
2023	11242.88	10845.83	11666.01	10648.91	11890.16	10280.27	12181.28

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